

$\mathcal{L} + \frac{1}{\mathcal{L}} = \text{special case}$

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if, $x + \frac{1}{x} = \sqrt{3}$ तो $x^6 + 1 = 0$ या $x^6 = -1$

Concept:

$$x + \frac{1}{x} = \sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = \sqrt{3}^3 - 3 \times \sqrt{3} = 0$$

$$\Rightarrow \frac{x^6 + 1}{x^3} = 0$$

$$\Rightarrow x^6 + 1 = 0 \text{ या } x^6 = -1$$



if, $x + \frac{1}{x} = \sqrt{3} \rightarrow x^6 + 1 = 0$ या $x^6 = -1$

Concept:

यदि, $x + \frac{1}{x} = \sqrt{3} \Rightarrow x^6 = -1$

$x^6 = x^6 \times x^0 = (x^6)^1 \times x^0 = x^6 = -1$

तो $x^{29} + x^{23} + x^{15} + x^9 + x^6 + 4 = ?$

$x^{61} + \frac{1}{x^{61}} = x + \frac{1}{x} = \sqrt{3}$

~~$x^{23} \cdot (x^6 + 1) + x^9 \cdot (x^6 + 1) + x^6 + 4 = 3$~~

$(-1)^{\text{odd}} = -1$ $(-1)^{\text{even}} = 1$



16. If $\left(x + \frac{1}{x} + 1\right) \left(x + \frac{1}{x} - 1\right) = 2$, then

($x > 0$)

यदि $\left(x + \frac{1}{x} + 1\right) \left(x + \frac{1}{x} - 1\right) = 2$ है, तो

A. $x^{38} + \frac{1}{x^{38}} = x^2 + \frac{1}{x^2} = \sqrt{3^2 - 2} = 1$

B. $x^{53} + \frac{1}{x^{53}} = ? - \frac{1}{x} - x = -\left(x + \frac{1}{x}\right) = -\sqrt{3}$

C. $x^{75} + \frac{1}{x^{75}} = x^3 + \frac{1}{x^3} = \sqrt{3^3 - 3\sqrt{3}} = 0$

D. ~~$x^{320} + x^{314} + x^{250} + x^{244} + x^{94} + x^{88} + x^{45} + x^{39} + x^6 + 5 = ?$~~ ④

$$\left(x + \frac{1}{x}\right)^2 - 1^2 = 2$$

If, $\left(x + \frac{1}{x}\right) = \sqrt{3} \Rightarrow x^6 + 1 = 0$ या $x^6 = -1$

$$x^{38} = (x^6)^6 \times x^2 = (-1)^6 \times x^2 = x^2$$

$$x^{53} = \frac{x^{54}}{x} = \frac{(x^6)^9}{x} = \frac{1}{x}$$

$$x^{75} = (x^6)^{12} \times x^3 = x^3$$

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If,

$$x + \frac{1}{x} = 2 \quad \text{तो,} \quad x = 1$$

x=

$$x + \frac{1}{x} = 2$$

$$x^2 + 1 = 2x$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0 \Rightarrow x-1=0 \\ x=1$$

CONCEPT

If,

$$x + \frac{1}{x} = -2 \quad \text{तो,} \quad x = -1$$




$$\text{if, } x + \frac{1}{x} = 2$$

$$\Rightarrow x = 1$$

$$\text{तो } x^3 + 5x^5 - 7x^2 + 4 = ?$$

$$1 + 5 - 7 + 4$$

$$= 3$$


18. If $m + \frac{1}{m-3} = 5$, then $(m-3)^8 + \frac{1}{(m-3)^8} = ?$

यदि $m + \frac{1}{m-3} = 5$ है, तो $(m-3)^8 + \frac{1}{(m-3)^8} = ?$

[A] 2

[B] 0

[C] -2

[D] 15

$$= 1 + \frac{1}{1} = 2$$

$$(m-3) + \frac{1}{m-3} = 2$$

$$\Rightarrow m-3=1$$

$$m=4$$

19. If $K + \frac{1}{K} + 2 = 0$ and $K < 0$, then what is the value of $K^{17} + \frac{1}{K^{11}}$?

यदि $K + \frac{1}{K} + 2 = 0$ और $K < 0$, तो $K^{17} + \frac{1}{K^{11}}$ का मान क्या है?

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[A] -17

[C] -1

☒ [B] -2

[D] 0

$$-1 - 1 = -2$$

$$K + \frac{1}{K} = -2$$

$$\Rightarrow K = -1$$

20. If $x + \frac{1}{x} = -2$, then what is the value of $x^7 + x^{-7} + x^2 + x^{-2}$? ($x < 0$)

यदि $x + \frac{1}{x} = -2$, तो $x^7 + x^{-7} + x^2 + x^{-2}$ का मान क्या है? ($x < 0$)

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[A] 4

[B] 2

[C] 1

[D] 0

$$\Rightarrow x = -1$$

21. **If** $x + \frac{1}{x+5} = -7$ **then** $(x+5)^{19} - \frac{1}{(x+7)^{28}} = ?$

यदि $x + \frac{1}{x+5} = -7$ है तो $(x+5)^{19} - \frac{1}{(x+7)^{28}} = ?$

[A] 0

[C] -2

[B] 2

[D] 1

$$(x+5) + \frac{1}{(x+5)} = -7$$

$$\Rightarrow x+5 = -1$$

$$x = -6$$

$$(-1)^{19} - 1 = -2$$

22. If $x + \frac{1}{x} = -2$, then $x^{2n+1} + \frac{1}{x^{2n+4}} = ?$ (where n is an integer)

यदि $x + \frac{1}{x} = -2$ है, तो $x^{2n+1} + \frac{1}{x^{2n+4}} = ?$ (जहाँ n एक पूर्णांक है)

[A] 2

[B] 4

[C] -2

[D] 0 ✓

$$x = -1$$

$$\overset{\text{odd}}{(-1)} + \frac{1}{\overset{\text{even}}{(-1)}}$$

$$= -1 + 1 = 0$$

23. If $\frac{x}{5} + \frac{5}{x} = -2$, then what is the value of x^3 ?

यदि $\frac{x}{5} + \frac{5}{x} = -2$ है, तो x^3 का मान क्या है?

☒ [A] -125

[B] -25

[C] $\frac{1}{125}$

[D] 625

~~$\left(\frac{x}{5}\right) + \left(\frac{1}{x}\right) = -2$~~

$\Rightarrow \frac{x}{5} = -1$

$x = -5$

If, $x + \frac{1}{x} = 1$

या

$x^2 - x + 1 = 0$

तो, $x^3 = -1$

CONCEPT

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$$a^3 + b^3 = (a+b) \cdot (a^2 - ab + b^2)$$

I. $x + \frac{1}{x} = 1$

तो, $x^3 + \frac{1}{x^3} = 1 - 3x = -2$

$\Rightarrow x^3 = -1$

II. $x + \frac{1}{x} = 1$

$x^2 + 1 = x$

$(x+1) \cdot (x^2 - x + 1) = 0 \times (x+1)$

$\Rightarrow x^3 + 1 = 0$

$\Rightarrow x^3 = -1$

24. If $\frac{x}{y} + \frac{y}{x} = 1$ & $x + y = 2$, then the value of $x^3 + y^3$?

यदि $\frac{x}{y} + \frac{y}{x} = 1$ & $x + y = 2$, तो $x^3 + y^3$ का मान क्या होगा?

(CPO 2023)

[A] 0 ✓

[B] 1

[C] 3

[D] 2

$$\left(\frac{x}{y}\right) + \frac{1}{\left(\frac{x}{y}\right)} = 1$$

$$\Rightarrow \left(\frac{x}{y}\right)^3 = -1$$

$$x^3 = -y^3 \Rightarrow x^3 + y^3 = 0$$

25. If, $\frac{r}{13} + \frac{13}{r} = 1$ then the value of r^3 is:

यदि, $\frac{r}{13} + \frac{13}{r} = 1$ है, तो r^3 का मान ज्ञात कीजिए।

$\frac{r}{13} + \frac{13}{r} = 1$
(SSC CGL PRE 2022)

[A] -2157

[B] 2197

[C] 2157

✓ [D] -2197

$$\left(\frac{r}{13}\right)^3 = -1$$

$$r^3 = -2197$$

if,

$$x + \frac{1}{x} = -1$$

या

$$x^2 + x + 1 = 0$$

$$\text{तो } x^3 = 1$$

$x = \text{imaginary}$

$$x + \frac{1}{x} = -1$$

$$x^2 + 1 = -x$$

$$(x-1) \cdot (x^2 + x + 1) = 0 \times (x-1)$$

$$x^3 - 1 = 0 \Rightarrow x^3 = 1$$

26. If $\frac{a}{b} + \frac{b}{a} = -1$ and $a - b = 2$, then the value of $a^3 - b^3$ is:

यदि $\frac{a}{b} + \frac{b}{a} = -1$ और $a - b = 2$ है, तो $a^3 - b^3$ का मान ज्ञात करें।

(SSC CGL PRE 2022)

[A] 0 ✓

[C] 1

[B] $\frac{1}{2}$

[D] -1

$$\left(\frac{a}{b}\right)^3 = 1$$

$$a^3 = b^3$$



27. If $5x + \frac{1}{2x} = \sqrt{5}$, then find $50x^4 + \frac{1}{100x^4}$?

यदि $5x + \frac{1}{2x} = \sqrt{5}$ है, तो $50x^4 + \frac{1}{100x^4}$ ज्ञात कीजिये?

[A] -1

[B] -1/2

[C] -4/3

[D] -3/2

$$-\frac{1}{2} - 1 = -\frac{3}{2}$$

$$5x + \frac{1}{2x} = \sqrt{5} \quad \text{---} \quad \cancel{2 \times 5 \times \frac{1}{2}}$$

$\times 4x^2$

$$20x^3 + \frac{1}{2x} = 0$$

$$100x^4 + 1 = 0 \Rightarrow 100x^4 = -1$$

28. If $x^4 + y^4 = x^2y^2$, then find $x^6 + y^6$?

यदि $x^4 + y^4 = x^2y^2$, तो $x^6 + y^6$ ज्ञात करें?

(CPO 2023)

[A] 1

[C] 2

[B] 3

[D] 0

$$\begin{aligned}
 & (x^2)^3 + (y^2)^3 \\
 & \rightarrow (x^2 + y^2) \cdot \cancel{(x^4 + y^4 - x^2y^2)} \\
 & = 0
 \end{aligned}$$



CONCEPT

$$(x^2 + y^2)^2 = x^4 + y^4 + 2x^2y^2$$

$$(x^2 + y^2)^2 - (xy)^2 = x^4 + y^4 + x^2y^2$$

$$(x^2 + y^2 + xy) \cdot (x^2 + y^2 - xy) = x^4 + y^4 + x^2y^2$$

A (माना)

$$x^2 + y^2 + xy = A$$

$$x^2 + y^2 - xy = B$$

B (माना)

$$(x^2 + y^2) = \frac{A + B}{2}$$

$$xy = \frac{A - B}{2}$$